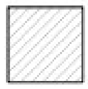
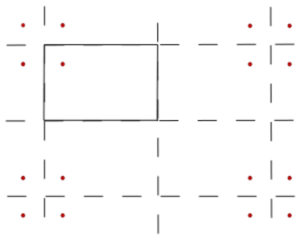
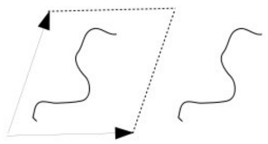
If a group G acts on a topological space X then X is partitioned into the orbits of the action $G \cdot x = \{g \cdot x; g \in G\} \subset X$, ie we have an equivalent relation $x \sim y \Leftrightarrow \exists g \in G : y = g \cdot x$.

The **quotient space** is $X/G = X/\sim$ which is the set of orbits or equivalence classes.
$U \subset X/\sim$ is open in the quotient topology if $\bigcup U \subset X$ is open.

$*2222$ acting on $\mathbb{R}^2$ quotient = fundamental domain = 

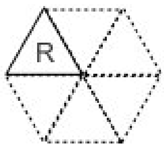


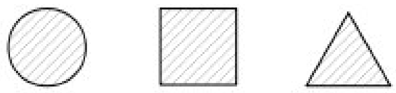
translation group $\circ$ acting on $\mathbb{R}^2$



quotient is a torus

$*333$ acting on $\mathbb{R}^2$

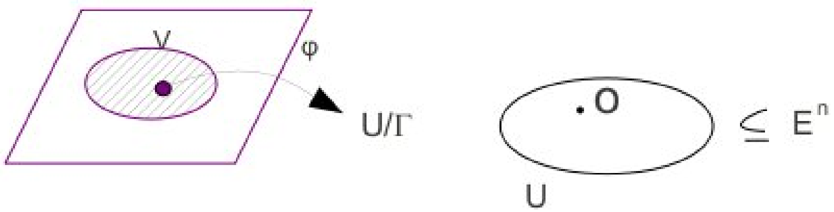




as a topological space a closed disk quotient

The quotient of $\mathbb{S}^2$ or $\mathbb{E}^2$ by any of our discrete groups will - as a topological space - by a surface (possible with boundary) But we want to keep track of slightly more information - for each point, what was the stabilizer of the action.

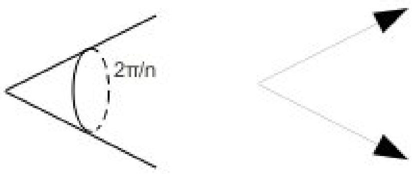
An orbifold is a space locally modelled on $\mathbb{E}^n$ modulo some group action



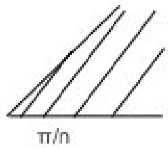
A 2-orbifold is a surface with boundary ($\Gamma = D_1$) and marked cone points (where $\Gamma = C_n \subset O_2$) and marked corner points ($\Gamma = D_n$).

$\mathbb{E}^2/D_1 =$

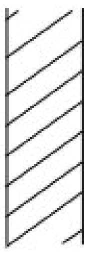
$\mathbb{E}^2/C_1 =$



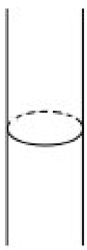
$\mathbb{E}^2/D_n =$



$\mathbb{E}^2/*\infty\infty =$



$\mathbb{E}^2/\infty\infty =$



All connected compact surfaces is a sphere with handles $\circ$ or crosscap x and with boundary components $*$.

$$\Sigma_{g,k} = \underbrace{\circ\circ...\circ}_{g}\underbrace{**...*}_{k}$$

$$N_{h,k} = \underbrace{xx...x}_{h}\underbrace{**...*}_{k}$$

Orbifold notation

$\circ$ handle

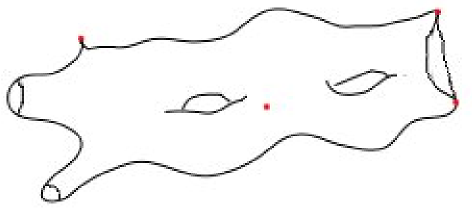
x crosscap

$*$ boundary component

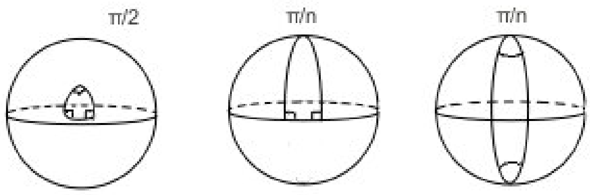
n (at the beginning): cone point

n (after a $*$): corner in that boundary

$23 \circ \circ * *22*$



quotient orbifold is a disk with corners in the boundary polygons with angles of the form $\frac{\pi}{n}$



$*2222$ $*235$
$*333$ $*234$
$*236$ $*233$
$*244$

$22n$
$*nn$

2222 235
333 234
236 233
244

$22n$
nn

$2*3$	$3*2$
$2*22$	$2*n$
$4*2$	

$\circ$ torus

xx Klein bottle

$**$ cylinder

$*x$ Möbius band

$22*$ open pillow case

$22x$ $\mathbb{R}P^2$ with cone points

nx $\mathbb{R}P^2$ with cone points